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Autoreport

THE ROLE OF EMOTIONS AS TRANSMITTED BY THE USE OF CARTOONS AND GAMES IN CONSTRUCTING MILIEU IN NEGOTIATING MATHEMATICAL KNOWLEDGE IN PRIMARY AND LOWER SECONDARY SCHOOLS

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Abstract

In this research I have studied the teaching/learning processes in lower secondary schools, with the aim of constructing particular teaching environments (milieu) and refining particular 'tools or attitudes', which the teacher can use in order to improve the understanding of mathematical topics. Considering that 'school learning operates by transforming conceptual thought' (Vigotskij, 1966; Chapter 4), the teacher's activity is connected to a logical perception, which is characteristic for each and every learner. Indeed, every pupil is endowed with 'simple' and 'elementary' ideas which characterise the initial phase of learning. When the teacher teaches, it must always be remembered that, in any given moment, something is changing in the learners' minds.

My ideas about teaching emphasizes places attention not only the teaching phase but also the possible repercussions in the learning phase.

The specific use of a game and particular tools or teachers' attitudes in our experimentation are favourable in mediating and negotiating mathematical meaning in class. It is important to underline that the word 'attitude' indicates the totality of all those gestures, words and fictitious strategies which the teacher can knowingly use in introducing various mathematical aims into a particular atmosphere, one which emotionally involves the learner; in most cases, the teacher's attitude is declared and made explicit throughout the teaching phase. An important aspect in communicating a mathematical aim will be to encourage the learner to take into consideration the problem proposed as their personal problem (devolution), to be able to identify mathematical tools in resolving problems. Devolution appeals to the motivation of learner (Spagnolo, 1998), who does not only have to accept the game proposed (a synonym for a situation) but they must research the best strategies which will let the learner to win.

The study of teaching activities has been dealt with from many points of view:

- from the dynamics of interaction between problem situations and the learner, teacher-learner, or between the learners themselves
- from the use of the teacher's attitudes and precise tools with which to mediate and negotiate meaning
- the learner's behaviour and
- the teacher's teaching strategies regarding the twofold role of teacher-tutor.

For this reason, as part of my activity as teacher-researcher, I believe it important to inquire, within the limits of scientific knowledge, how certain learning processes occur from a biological point of view. I also wondered what effectively the role of emotions in learning is and if it would be possible to use emotions not by 'imposing' knowledge but by encouraging learning to become a 'wish to learn' (*apetitus noscendi*, Changeux, 2003).

Aim

The context or environment has an essential function in negotiating meaning as means of communication in the teaching system. According to Chevallard and Johsua (1982), the context comprises three components: the teacher, learner and the knowledge to be taught. Subsequently, Cornu and Vergnioux (1992) introduced the concept of the noosphere, thereby considering the social context which involves and interacts with the teaching system. The use of various linguistic registers and personal, cultural experience are important elements in encouraging communication. Vygotskij has drawn on Piaget's assertion: "Being aware of an operation indeed means passing from an action to a linguistic plan; it means, therefore, inventing it in your imagination, to be able to express it in words" (Vygotskij, 1990, p.227). Clearly, in order to pass from one to the other, the action must be contextualised and then recognised and accepted buy the learner.

In order for understand and encourage the process of passing from an action to a linguistic plan in the teaching process, the connection between the 'word and its meaning' must be taken into consideration; this is not stable but subject to a process of evolution (Vygotskij, 1990, Chapter VIII).

My objective is to study the acceptance of this devolution, connecting it to affective learning in order to emphasise the tie between the word and image and their meaning to the learner.

This takes place between the meaning of the term in its internal language, common language (that is, everyday extra-curricular experience) and the specific meaning of mathematical terms.

Theoretical framework

In my experimentation, I have created cartoons and the 'Guess the Number' game. Guy Brousseau (1989a) defined the teaching concept of mathematics as science, 'a science which is concerned with the production and communication of mathematical knowledge and in what way this production and communication are specific'; this science has specific study aims:

- the essential operation of disseminating knowledge, the conditions of this dissemination and the transformations which it produces, both for knowledge and its users;
- the institutions and activities whose aim is to facilitate these operations.

One of the theoretical reference models in both cases is Guy Brousseau's Situation Theory, which is present in the choice of methodology in creating the situation/problem. The theoretical reference paradigm for this research is the systematic 'knowledge-learner-teacher' approach. Studies on possible relationships between these three regard the theory of learning research including content, methods and a paradigm. Knowing the research paradigm means possessing:

- a suitable language

- a suitable methodological tool and
- a suitable statistical tool

From the teacher's point of view, checking the tools for learning research means:

- having an idea about the positive role of mistakes
- constructing a research paradigm with which to resolve teaching problems and
- having autonomous ideas with which to communicate the process and result of the research

The research theory of learning is an important element for teachers and it includes: knowing about the research phases, constructing an a-didactic situation for the problem, formulating research hypotheses which are appropriate to the a-didactic situation, and producing documentation for those who do not know this activity.

A theoretical reference for analysing the teaching situation and learners' behaviour during the game (in addition to those of semiotic, epistemological, psychological and historical enquiry) is that of **neuroscience** (J. Pierre Changuex, 1998 e 2003; Antonio Damasio, 1995; Gerald Edelman, 1987). In both experimentations the a-didactic situation is based on a game and it was very interesting to study the parts of the brain, which are concerned with analysing effective learning with a game-like activity. Analysing the a-didactic situation with reference to mathematical cartoons takes into consideration the use of specific graphic tools, used in creating cartoons, which seem to facilitate mathematical communication. These tools are used in attempting to recreate an environment which is familiar and easily recognisable by the learners.

In conclusion, we can say that the tools used for analysing and constructing these environments are: neuroscience (J.P.Changeux, A.Damasio, G.Edelman), epistemology, Situation Theory (G.Brousseau), Semiotics (S.McCloud), psychology (L.S.Vygotskij) and Embodiment Mathematics (Lakoff & Nùñez). The components of this inquiry in the teacher-learner-knowledge environment are: culture, sociality, images, symbols, sensory-motor experience, motivation and affect in learning mathematics, the teacher's role and the teaching environment (milieu).

Realization of the aim

The guessing game 'Guess the number' was analysed and subdivided into specific phases when used as a teaching tool for negotiating between arithmetic and pre-algebraic meanings, and the iconic role of symbols. With this activity the passing from arithmetic and pre-algebraic language was analysed by starting with the problem, which uses a purely natural language.

In our experimentation the use of cartoons, instead is to verify and check the notions acquired or present in a class but their use is also provided for when introducing new, mathematical concepts. It was stimulating to analyse how a verbal language gradually develops and how its syntax and semantics become ever richer and more complex; this is also the case with gradually-developing graphic language. Commencing with a function which is predominantly narrative and pictorial, graphic language is integrated ever more deeply with its cognitive and communicative functions, becoming a most suitable 'environment' for the communication and learning of notions. Within a neuro-physiological and Vygotskian perspective, the teacher's role is fundamental as the mediator of this process. By creating or choosing appropriate designs and suitable contexts, they must try to stimulate interest and encourage social interaction. Indeed through these activities, children record many experiences of movement in which their 'cognitive unconscious' (*Lakoff & Nùñez, 2000*) is recalled, updated and made explicit. Regarding the multi-sensorial use of cartoons as linguistic mediators, the approach to reality is fundamentally far from being a simple connection between scholastic learning and everyday experience. In accordance with neuro-physiological studies, the recent theory of Embodied Mathematics (*Lakoff & Nùñez, 2000*) and the results obtained from my experimentation, I have sought to understand how certain scientific knowledge within the learners' minds is constituted, considering that mathematical concepts, logical structures and tools used are the basis of a real bodily experience (in the case of cartoons, it is 'virtual').

Research Questions and Hypotheses

The choice of carrying out experimental research which includes games or cartoons does not arise only from the necessity of investigating concepts about learners, rather from the need to suggest a new way of 'doing' mathematics, which appeals to a motivational state as regards personal needs. Various objectives have guided me in the selection of teaching tools which are to be used in structuring my experimentation. Some of these are:

- studying multi-sensorial aspects in teaching and learning activities for mathematics
- analysing the game-like characteristics of mathematics in relation to the motivation and interest of doing this type of mathematical activity (*appetitus noscendi*, Changuex, 2003)
- developing a real sensitivity in the learners in interpreting and comprehending symbolic images
- organising a grammar which is the most characteristic possible in creating and interpreting a mathematical cartoon

- analysing, from a neuro-physiological point of view, the use of parallel and serial thought by means of diagrams
- analysing the role and meaning of the graphic tools, used in creating cartoons, for students which are recognised by the cartoon's iconic code or those which have been introduced *ad hoc* by the teacher (the teacher's implicit tools)
- analysing the problem of mathematical communication in multi-cultural environments.

Regarding the 'Guess the number' game, my objectives are

- thoroughly analysing the relationship between natural and symbolic language
- analysing how the constructing of patterns intervenes in the process of anticipation

We can, therefore, outline the following **research hypotheses**:

H1 constructing teaching situations, involving a conscious use of cartoons to facilitate devolution

H2 constructing teaching situations, involving a conscious use of arithmetic games to facilitate devolution

H3 constructing learning/teaching milieu which encourage an instrumental use of functional emotions as regards mathematical knowledge (from the learner's point of view)

H4 constructing learning/teaching milieu which encourage an instrumental use of functional emotions as regards mathematical knowledge (from the teacher's point of view).

Experimentations

First Experimentation: "Clamat's Cartoons"

The sample

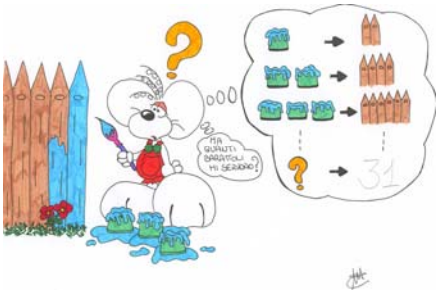
The experimentation was carried out in three classes in two secondary schools in Palermo (Italy) and two classes in Ficarazzi, a village near Palermo. The students in the sample were from between 10 - 13 years old.

Methodology: instructions and organizing the cartoons

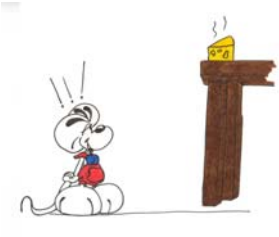
Four cartoons were deployed in the experimentation, being organized according to the various teaching topics.

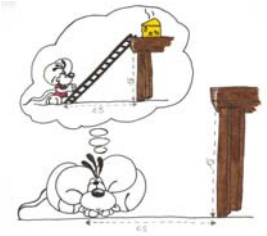
A teaching unit was prepared for each cartoon thus:

- The problem of the pots

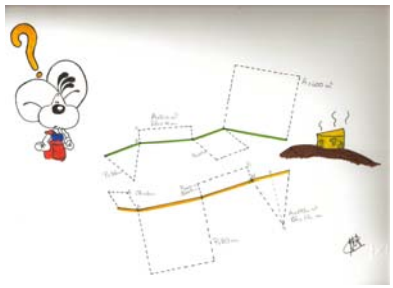


- The problem of the ladders





- The problem of the roads



- The problem of the house

Structure

One of the key aims of Clamat's cartoon is to provide a framework within which students can establish relationships and meaning, similar to an index system in a library. This allows concepts to be filed away in compartments in the memory, thereby making their retrieval much easier. Much

information can be contained within a small space and the ideas can be arranged in such a way so as to identify relationships between concepts and encourage thinking from a larger perspective.

Introduction to the Activities

The activities found in Clamat's cartoons can be easily integrated into lesson plans and those described below provide an excellent opportunity for students to have an overview of a topic, in short, to glimpse the bigger picture. These activities will also help learners to make the all-important connections, which establish meaning.

The main character to be found in these activities is Clamat, a friendly mouse. He is prone to everyday problems and he like to talk about these.

Instructions

1 This activity can be introduced by demonstrating one of Clamat's cartoons to the class. This simplest and probably most effective activity is to have a class discussion about Clamat's problems.

2 The learners receive two photocopies: one with the cartoon strip and the other a guided questionnaire.

3 The teacher asks the learners to answer the following questions from the questionnaire:

1. What is Clamat's problem?
2. What are the data in the problem?
3. What strategies could Clamat use in resolving the problem?
4. Why have you chosen these strategies?
5. What does the message in the bubble mean?
6. What do the arrows represent?

Second Experimentation: "Guess the Number"

With the aim of making a model of researcher-teacher activity, the activity is organised in the following way:

- **initial aim**: to solve the problem
- **condition which justifies modelling**: not knowing how to solve the problem
- **discursive phase**: the teacher listens to individual learners' comments and ideas

- **translation phase**: the teacher translates the problem described in natural language into mathematical symbols and then begins to reread it and reflect on the above
- **a mathematical model is created and the problem of this model is reread**: in this way the use of mathematical tools for solving the problem is highlighted
- **the mathematical problem in the model is solved**
- **go back to comment on the solution of the real problem**: the relationship between the model and reality can be exploited in interpreting the mathematically-obtained result

The sample

Our experimentation regards the first, second and third classes in a lower secondary school. The learners are 11-13 years old and on average there were 22 learners in each class. In the first and second classes, the aim of the experimentation was to introduce symbolic language, starting with natural language; in the third class, the aim was to lead to reflecting about the use of pre-algebraic language which the teacher consciously used after having introduced the concept of identity and equations.

Methodology Introduction to the Activities

After having subdivided the learners into groups, the teacher explained that the aim of the game was to work out how the teacher could guess a number thought of by the learners, who then received some specific information about the game:

- multiply the number thought of by 5
- add 6
- multiply this sum by 4
- add 9 to the total
- multiply the final number by 5
- write down the result

Once the game has finished, the teacher can hand out the following questionnaire:

- Describe each step of the procedures, followed in arriving at the solution
- Comment on the meaning of the formula which you used in the 1st phase
- Comment on your solving strategies: what is different about the strategies used in solving the game?
- Does changing the positions of the symbols in the result change anything?

Conclusion

About the first experimentation, the most generally widespread idea about the language of cartoons is that it is a juxtaposition of a language of 'words' with a language of 'images'. The fact is that, even if we are dealing with a 'similar' and simple juxtaposition, the global effect would probably be not of words taken in themselves but their relationships (Barberi, p.203). As with reading a written text, learning to read a cartoon is the arrival point of a long process of symbolisation which, by design and reading the design, is transformed into definitive reading. Therefore, reading has as its presuppositions not so much in the de-coding but in the habit of treating a sense of images and signs in the reconstruction of a story.

Our experimentation has shown that learning can be integrated into the mathematical subject to be described, thanks to a strong emotional connotation and motivation, which is turn is bound up with the learner's will to continue playing and win by having fun. My experimentations has helped me to verify that the mental processes and perceptive mechanisms are not spontaneous and natural at all but they must be decoded, the competence of which depends on various educational and cultural variables. These mental processes and perceptive mechanisms are required in gathering the information and meaningful connections contained in an illustration (or in a symbolic expression) and in selecting and organising visual stimuli according to determined criteria (similarity/differences, figures/background etc). It can be inferred from the results of my experimentation that interpreting symbols is not so much tied to recognising the object described and the immediate global approach that one can have with that image, rather the skill in penetrating details in the figure, putting together single features by question and observations.

Emotions are indispensable for creating a memory because they organise the memory into a sequence of events; this establishes the importance of emotions. A sense of time and order are essential because a memory is considered as such and not a thought or a vision without any relation to past events. But a memory does not exist without a context. With our experimentation we have seen the importance of using a game or a cartoon, aware of the fact that cultural and corporal characteristics of the class may be similar. Thus we can hope that the mathematical message transmitted by a real situation-problem, described by a cartoon or created by the teacher in a game, is the same for each learner. Each learner must be able to recognise the context and know how to move inside of it. In this way and if we know the real expression of mathematical enquiry, learning can be stimulating.

Below I would like to explain my results with reference to my research hypotheses H1, H2, H3 and H4:

1. Regarding hypotheses H3 and H4, the teacher used tools (attitudes, words, gestures, icons, boardwork……..) in the game and cartoons, both of which contributed to developing and understanding various specific concepts in the teaching aim. In this way, the symbols became tools of mediation, signs with their own specific meaning, which the teacher brought into the classroom as external objects to be studied, beginning with their close relationships with a 'sign', which lives and has meaning in a natural language.
2. Regarding the 'Guess the number' game, the board is a very powerful tool of mediation for the teacher since, if well analysed, it leads to a conscious and critical reflection about the use of mathematical tools and the generalisation of analysable problems by using a formula. With its model-like nature, the game infers checking and verifying phases of the activity and a comparison of knowledge acquired with a single (initial) visual experience.
3. Discursive processes (developed throughout the validation phase, appropriately organised in the two experimentations under the teacher's careful mediation in an atmosphere which emotionally involved the learner) had a consciousness-raising role (according to Vygotskij, one of the essential characteristics of 'scientific concepts') and they generally contributed to developing competence in mathematical concepts.
4. In discussing my research hypotheses, it was interesting to observe that passing from common to mathematical language, the learners used a language which was tightly bound up with their own cultural and social experience. This language not only puts natural language in relation to mathematical language but also the concepts which can arise from daily life. These ideas often involve mathematical objects, creating misconceptions (we can think about the notion of 0 and its daily use in common language). Through my experimentation, it was possible to notice and analyse various jumps in logic tied to common experience whether analysing each learner's personal interpretation for each cartoon or their active participation during the translation phase of the 'Guess the number' game.
5. From a multi-cultural point of view, it is important to highlight that the use of cartoons proved to be an excellent communicative tool. Thanks to their iconic code, cartoons evoke lived experiences which diminished linguistic differences on entering and exiting the cartoon. At the end of the exercises, the learners not only understood the problem, which they attempted to solve, but they produced a purely personal iconic code for communicating logical procedures, the meanings of some calculations and even the organisation of some of the same data in the problem

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