TEACHING OF ANALYTIC GEOMETRY
AND VECTOR CALCULUS AND PROPOSALS
OF PROBLEMS’ SOLUTIONS

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Abstract. The aim of our work was to describe situation in teaching of analytic geometry and vector calculus at secondary schools in the Slovak Republic, to specify possible students’ problems those are due to the application of methods of analytic geometry and vector calculus in various geometric tasks and to propose their possible solutions. The aim was realized in three parts: theoretical, experimental and applications. Following the conclusions of experimental part we proposed new strategies in analytic geometry and vector algebra that can help to eliminate the students' problems in solving mathematical problems not only from analytic geometry and vector algebra.

Résumé. Le but de notre travail était décire la situation de l’enseignement de la géométrie analytique et du calcul vectoriel aux écoles secondaires en Slovaquie, spécifier des problèmes possibles que cause l’application des méthodes de la géométrie analytique et du calcul vectoriel dans différents exercices géométriques et proposer leur possibles solutions. Les objectifs du travail on a réalisé dans trois phases: théorique, expérimentale et applique. En raison de nos résultats implicites de la phase expérimentale on a proposé dans la géométrie analytique et dans le calcul vectoriel des stratégies nouvelles qui peuvent éliminer des problèmes des étudiants résoudre des exercices différents mathématiques, non seulement de la géométrie analytique et du calcul vectoriel.


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Probleme der Schüler beim Lösen verschiedener mathematischer Aufgaben, nicht nur aus der analytischen Geometrie und der Vektorrechnung, behilflich seien.

Riassunto. Obiettivo del nostro lavoro è stato quello di descrivere la situazione dell’insegnamento della geometria analitica e del calcolo vettoriale nelle scuole secondarie della Repubblica Slovacca, per individuare e precisare quei possibili problemi dovuti all’applicazione dei metodi della geometria analitica e del calcolo vettoriale nei vari “scopi” che può avere la geometria, quindi di proporre le possibili soluzioni. Il tutto è stato realizzato in tre parti: teorica, sperimentale e applicativa. Seguendo le conclusioni della parte sperimentale abbiamo proposto nuove strategie per la geometria analitica e per l’algebra vettoriale che può aiutare ad eliminare le difficoltà degli studenti nel risolvere problemi matematici non soltanto di geometria analitica ed algebra vettoriale.

Abstrakt. Cieľom našej práce bolo popísať stav vyučovania analytickej geometrie a vektoroveho počtu na strednej škole v Slovenskej republike, špecifikovať možné problémy žiakov s aplikáciou metód analytickej geometrie a vektoroveho počtu v rôznych geometrických úlohách a navrhnuť ich možné riešenie. Ciele práce sme realizovali v troch fázach: teoretickej, experimentálnej a aplikačnej. Na základe výsledkov vyplývajúcich z experimentálnej časti práce sme navrhli nové stratégie v analytickej geometrii a vektorovej algebře, ktoré môžu napomôcť odstráneniu problémov žiakov v riešení rôznych matematických úloh, nielen úloh z analytickej geometrie a vektorovej algebry.

Key words: analytic geometry, vector calculus, theory of didactic situations, actual situation of teaching of geometry in Slovak schools, problems in teaching of analytic geometry and vector calculus, solving of those problems, statistical software C.H.I.C.

1 Introduction

“Two millennium ago geometry was a perfect and unique mathematical discipline which was founded on the axiomatic base. It seems nowadays that among all of classical parts of mathematics only geometry isn’t axiomatic”. Even though this statement of English mathematician Zeemann is dated in 1970, its actuality and analogy with recent teaching of mathematics at secondary school is amazing. As since it elapsed more than three decades, the new millennium did not bring the remarkable changes in teaching of mathematics. Henceforth, geometry belongs to the most problematic parts of secondary school mathematics, students are not able to experiment and to utilize their knowledge from the different parts of mathematics in solving the geometrical problems, they prefer arithmetic and algebraic approach (methods of calculus in geometry) and methods of construction in comparison with methods of analytic geometry and vector calculus, their space imagination is not sufficiently developed. These facts lead us to focus our attention on the teaching of geometry, especially on analytic geometry and vector calculus at secondary schools.
We chose the area of analytic geometry and vector calculus also by the reason of understatement the methods that this area proposes for solving the geometrical tasks from secondary school mathematics. From our point of view the teaching of analytic geometry and vector calculus at secondary schools is practiced deficiently, isolated from the other parts of mathematics even thought analytic geometry is one of mathematical discipline that supports the formation of student’s thinking. In the same manner is also underestimated, in our opinion, the importance of vector calculus that was based on ideas from mechanics, optics and geography and from the other different areas. Even thought the vector calculus at secondary schools is only auxiliary tool for teaching the analytic geometry, the teachers are oblivious to possibilities of its separated application in solving some geometrical tasks.

Mentioned facts as well as our positive experience with teaching of vector calculus and analytic geometry by methods different from classical methods of Slovak educational system (methods from French secondary school teaching) formed our ideas to improve the teaching of analytic geometry and vector calculus at secondary schools in the Slovak Republic.

The aim of our work was to describe situation in teaching of analytic geometry and vector calculus at secondary schools in the Slovak Republic, to specify possible problems those are due to the application of methods of analytic geometry and vector calculus in various geometric tasks and to propose their possible solution. This aim we realized in following phases:

1. Historical and epistemological development of analytic geometry and vector calculus.
2. Survey of actual situation and historical background of teaching of analytic geometry and vector calculus considering textbooks used in Slovakia.
3. Analysis of actual mathematics French textbooks of secondary schools and their comparison with Slovak textbooks.
4. Actual situation and specification of problems in the teaching of analytic geometry and vector calculus at secondary schools.
5. Proposals of possible problems’ solutions and compilation of tasks’ collection those are leading to enhancement of teaching of analytic geometry and vector calculus at Slovak secondary schools with specific recommended strategies of their solution.
6. Verification of the selected tasks’ efficiency from our tasks’ collection by comparing the abilities and capacities of students that took part in experimental teaching (with using of our propositions and strategies) and students that participated in the teaching with using of methods and processes, which are typical for Slovak secondary schools.
2 Theoretical framework

Analyse of particular didactical problem in educational process from different aspects is a research objective for many didactic schools in the world. One of them is the French didactic school represented by Guy Brousseau, Yves Chevallard and Anna Sierpinska. Research base of the theory of didactic situations issued from this didactic school is analysis of problem in particular levels of didactic situations (Brousseau, 1998; Chevallard, 1992; Sierpinska, 2001).

Brousseau (1998) accepts the didactic situation like a situation for which is possible to describe the social intention of acquirement of student’s knowledge. This situation is realised in system called the didactic system (didactic triangle) that is composed from three subsystems: learner (student), learning (teacher), information and from relations between them. The relations – the didactic contract is possible defined like a results of negotiation (intervention) explicitly or implicitly defined relations between students or student’s group. It is the environment and the educational system that prepare the students to accept completed or nascent knowledge. They are exactly the rules of game to activate the student.

The basic notion of theory of didactic situation is the didactic environment (Brousseau, 1990). Following the Piaget’s theory the environment is source of contradictions and non-steady states of learner (subject) by process of adaptation (by Brousseau (1986) it is assimilation and accommodation). The environment is specific for each of knowledge.

Interactions between the subject (student) and the environment form particular levels of didactic situations with corresponding didactic environments. In our work we used the structure of didactic situations and environments by Margolinas (1994).

3 Realization of research

We realized research aims in three following phases:

Theoretical phase includes detailed study of historical background of origin of analytic geometry and of introduction of vector calculus. We emphasized the epistemological point of view of their development and we studied the different approaches in teaching of analytic geometry and vector calculus at secondary schools following the actual Slovak textbooks and selected older textbooks and in accordance with works’ aims we analyzed also actual secondary French textbooks. The objective of theoretical phase was realized the first three aims of work.

Experimental phase of work is represented by research (fourth aim of work), by study of pedagogical papers (curriculums, programs, textbooks) and also by analysis a-priori of possible research results (in accordance with theory
of didactic situations). This analyze allowed us to formulate following research hypothesis:

H1 Students’ capability in solving the open problems and in „problem solving“ is developed in the schools’ practice deficiently; the students are using mainly acquired abilities and algorithms in solving standard problems.

H2 The effective combination and using of knowledge from different area of mathematics in solving a particular geometrical problem make difficulties for the students.

H3 The students prefer arithmetic and algebraic approach (methods of calculus in geometry) in comparison with methods of analytic geometry and vector calculus in solving the geometrical problems.

H4 Students’ access to analytic geometry is isolated from the other parts of schools’ mathematics; vector calculus represents for students only a mathematical tool for analytic geometry.

H5 The teaching of geometry in the space has a decreasing trend in regard to deficient students’ motivation and their attitude towards tasks situated in the space.

Application phase includes some suggestions and propositions how to solve problems that were found during research. So we completed the collection of tasks from analytic geometry and vector calculus that are applying in different parts of mathematics. The most important point of this application phase was experimental verification of the efficiency of these tasks. In this sense we verified the validity of following hypothesis:

H: Application of methods and strategies of analytic geometry and vector calculus typical for French secondary school teaching will lead to better results of students at the end of our experimental teaching (in didactic test – students’ papers) in comparison with students who know only methods and strategies typical for Slovak secondary school teaching.

4 EXPERIMENTAL PHASE OF WORK

In accordance with formulated aim of research (fourth aim of work) we described actual situation of the teaching of analytic geometry and vector calculus at secondary schools in the Slovak Republic and specified possible problems that the students have in using the methods of analytic geometry and vector calculus in solving the different geometric problems. We realized this aim by research during two years (April 2002 – October 2004) with 109 students that were selected in six different groups:
A. group consisted of 7 students (18 years old) from the 4th class of business college.

B. group was formed by 9 students (17 years old) from the 3rd class 4-yearlong grammar school.

C. group included 28 students (17 years old) from the 7th class of 8-yearlong grammar school.

D. group consisted of 12 students (18 years old) from the last class of 5-yearlong scientific grammar school in Palermo.

E. group included 14 students from 2nd class of university (they were the future teachers of mathematics and biology).

F. group was formed by 39 students (16 – 17 years old) from the 3rd class of bilingual Slovak-French section of 5-yearlong grammar school.

The first part of research was realised by students paper with two tasks:

<table>
<thead>
<tr>
<th>Task 1</th>
<th>Let’s have given equilateral triangle $ABC$ with coordinates of vertexes $A[−\sqrt{3}, 2], B[3\sqrt{3}, 2]$ and with orthocenter $V[\sqrt{3}, 0]$. Determine the coordinates of the vertex $C$.</th>
</tr>
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<tbody>
<tr>
<td>Task 2</td>
<td>Let’s have given cube $ABCDEFGH$ with edge 2 unites long. We note $\vec{a} = \overrightarrow{AB}, \vec{b} = \overrightarrow{DH}, \vec{c} = \overrightarrow{FG}$. Calculate $(\vec{a} + 2\vec{b}) \cdot 3\vec{c}$.</td>
</tr>
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In the second part of our research we made analysis of students’ papers in accordance with theory of didactic situations, we emphasized the detailed analyze a-priori mainly from students’ part. Tasks selection and analysis of their solutions produced 20 possible strategies in three different mathematical frames for task 1 (arithmetic-algebraic, vector, analytical frames) and 5 strategies in three mathematical frames for task 2 (arithmetic-algebraic, analytical, geometrical frames). Each of strategy was describe by a few steps of solving that in statistical interpretation represent the didactic variables.

The main method of research work was the content analysis of students’ papers (Kerlinger, 1972) that may be realize in quantitative or non-quantitative way. In our work we present the content analysis of students’ papers with using of statistical software C.H.I.C. (Classification Hiérarchique Implicative et Cohésitive)\(^2\) that works with the frequencies of particular significant unites. In our case the role of significant unites had the particular strategies of solving for each of tasks – task 1 and task 2 that was grouped in the analytical categories – mathematical frames. The statistical software C.H.I.C. follows the modern trend in research as

it focuses on relations between some analytical categories (so not only in
determination the frequencies of isolated categories). So this software represents
the connection between quantitative and qualitative analysis. The statistical soft-
ware C.H.I.C. makes possible to compare the similarity of didactic variables pre-
sent in research, suggest the relations of coherence between these variables and
describes also the probability of realized implication between variables by prob-
ability rate of their realization. Apart from the relations between particular didac-
tic variables this software allows also the comparison of relations between whole
classes of didactic variables in three type of graph: similarity tree, implicative tree
and implicative graph. As input information for analyze of students’ papers we used
presence or absence of particular steps of all strategies realized in students’ works that
could have the value 1 (presence) or 0 (absence). The content analysis was realized
for each task and the statistical sample included 109 students. We completed this
analysis of research by statistical software C.H.I.C. also with interpretation and
explication of results concerning the character of students’ groups.

4.1 CONCLUSION OF EXPERIMENTAL PHASE

The interpretation of research by the help of statistical software C.H.I.C.
and by content analysis of students’ papers in accordance with the different
character of students’ groups allow us to verify the formulated hypothesis H1 –
H5 (see “The Realization Of Research”):

The hypothesis H1 (Students’ capability in solving the open problems and
in „problem solving“ is developed deficiently in the schools’ practices, the stu-
dents are using mainly acquired abilities and algorithms in solving the prob-
lems) was confirmed by solutions of students’ groups B and F (see the experi-
mental students’ groups). These students applied mainly the strategies of an-
alytic geometry and vector calculus because they took part in research in time
when they took lessons of this part of mathematics in their schools. On the con-
trary, the students of group C (which also took lessons of analytic geometry and
vector calculus during the realization of research) practiced the different strate-
gies in solving of both tasks. This students’ group and also the group E (stu-
dents at university which should know whole secondary mathematics) had the
least problems in task solving.

We realized the verification of hypothesis H2 (The effective combination
and using of knowledge from different area of mathematics in solving of par-
ticular geometrical problem make difficulties for the students) by the help of
task 2 that requests the effective using of vector calculus, space geometry and
also the methods of calculus in geometry. This hypothesis H2 was confirmed by
students’ papers of group A (they did not solve this task) and the group F (only
half of students tried to solve task 2). The best results in solving of task 2 achieved the students that should have the complete knowledge of secondary mathematics, so the students of groups D and E. However, also the students of group C showed their ability to combine the knowledge from different parts of mathematics in solving a geometrical problem. This result does not confirm the hypothesis H2 but it indicates the possible solving problems (the students of 8-yearlong secondary grammar school showed to be the least problematic group).

The conclusions obtained from interpretation of task 1 confirm also the hypothesis H3 (The students prefer arithmetic and algebraic approach (methods of calculus in geometry) in comparison with methods of analytic geometry and vector calculus in solving the geometrical problems) because the students applied mainly the strategies from arithmetic-algebraic frame (methods of calculus in geometry) even thought this strategies showed to be non-effective in task solving. Comparable number of solutions in vector frame to the number of solutions in arithmetic-algebraic frame is possible to explain by big number of students that learned analytic geometry and vector calculus during the realization of research (groups B, C and F that represent 69,7 % students of all groups).

We can verify the hypothesis H4 (Students’ access to analytic geometry is isolated from the other parts of schools’ mathematics, vector calculus represents for students only a mathematical tool for analytic geometry) by the preference of strategies from analytic frame that were applied in solving of task 1 (the general equation of line; minimum of students used parametric or another equation of line) and also by rare application of vector calculus (for example the scalar product) in analytic geometry (only the students of group F).

The last hypothesis H5 (The teaching of geometry in the space has a decreasing trend in regard to deficient students’ motivation and their attitude towards tasks situated in the space) was confirmed by bigger number of students that solved task 1 (98) in comparison with students that solved task 2 situated in space (76).

5 APPLICATION PHASE

H: Application of methods and strategies of analytic geometry and vector calculus typical for French secondary school teaching will lead to better results for students at the end of our experimental teaching (in didactical test – students’ papers) in comparison with students who know only methods and strategies typical for Slovak secondary school teaching.
Following the conclusions formulated in the experimental phase we completed tasks’ collection from analytic geometry and vector calculus with application of different parts of mathematics. Intention of this tasks’ collection was to propose to the teachers the collection of tasks with solutions by strategies from different mathematic frames and to underline the importance of using of vector calculus in various mathematic problems. We verified the efficiency of the tasks from our tasks’ collection of analytic geometry and vector calculus in experimental teaching. Concerning our positive experiences in teaching of mathematics following French textbooks we formulated hypothesis \( H \) mentioned above.

As method of work we used an experimental method (Gavora, 1997) in which we have two equivalent students’ groups and in each of them we applied an experimental change (experimental group) in comparison with the second group (control group). Process and experimental conditions excepting experimental change were identical in both groups. Results of experiment were worked out by statistical induction, which permits to generalize results obtained from research of selected set to whole population. As a method of statistical work we chose verification of hypothesis by \( t \) test (Kerlinger, 1972).

In experimental teaching (June 2004) we compared products of students from the 3rd class of grammar school that took part in teaching with typical Slovak methods and strategies (group \( A \) – 28 participants) and students that participated in experimental teaching with French methods and strategies (group \( B \) – 31 participants). In each of groups we taught 4 lessons - in the group \( A \) (control group) we taught by the help of strategies typical for Slovak secondary schools and in the group \( B \) (experimental group) we preferred the strategies from French secondary schools. On the basis of students’ papers that students worked out individually at the last lesson we compared efficiency of particular strategies. Results obtained in our experiment are in favour of teaching by tasks and strategies from our tasks’ collection, so by tasks those come from French secondary schools.

### 6 Conclusion

In introduction of article we determined the aim of our work - to describe situation of teaching of analytic geometry and vector calculus at secondary schools in the Slovak Republic, to specify possible problems, those are due to the application of methods of analytic geometry and vector calculus in various geometry tasks and to propose their possible solution. We defined this aim in more detail in six points those realization we describe in following analysis:
1. **Historical and epistemological development of analytic geometry and vector calculus.**

Importance of working-out this part have been, first of all, in inquiring the background of students’ problems with application of methods and processes in analytic geometry and with using of the vector in the geometric problems at secondary school, by reason of analogy between ontogenetic formation of everyone individual, which is consequence of evolutorial development of the world and development of our mathematical thinking with historical evolution of mathematics as a discipline.

2. **Survey of the actual situation and historical background of teaching of analytic geometry and vector calculus following textbooks used in Slovakia.**

Progressive introduction of methods of analytic geometry in solving the mathematical problems, the definition of vector and the constitution of analytic geometry on vector ground based in the history, result in ambition to integrate this part of mathematics in the teaching at secondary schools and colleges. Teaching the analytic geometry and vector calculus at secondary schools progressed, what indicate using of different methods and processes in mathematics’ textbooks which were edited during the development of Slovak educational system. The analysis of these textbooks included with analysis of actual textbooks used at Slovak secondary schools permit us to understand the reliable methods and practices in teaching of analytic geometry and vector calculus at secondary schools.

3. **Analysis of the actual mathematics French textbooks of secondary schools and their comparison with Slovak textbooks.**

Concerning our positive pedagogical experiences with teaching of analytic geometry and vector calculus following the French textbooks (at French-Slovak bilingual secondary school), we proposed to solve problems in analytic geometry and vector calculus at Slovak secondary schools by application of some strategies from French mathematics which prefer geometrical approach to problem solving.

4. **Actual situation and specification of problems in the teaching of analytic geometry and vector calculus at secondary schools.**

As the method of work we chose the content analysis of students’ works (109 participants), which consisted of two tasks (one was situated in the plane and the second in the space). Our research was realized following analysis of possible students’ solutions in accordance with *theory of didactic situations* (Brousseau, 1986, 1998, Chevallard, 1989, Margolinas, 1994, Sarazy, 1995, Sierpinska, 2001) and by analysis of historical and epistemological development of analytic geometry and vector calculus (the first part of our work) and analysis of secondary schools’ textbooks (the second part of our work). We described the research results by the help of statistical software C.H.I.C. (Gras et al. 2003).
The principle aim of our research was to verify hypothesis $H1 – H5$ (listed above).

We resumed the results of our research in following statements which are at the same time verification of our hypothesis and which explain in more detail the problems in teaching of analytic geometry and vector calculus at secondary schools (Regecová, 2003):

- Teaching the vector calculus and analytic geometry at secondary schools is separated from parts of geometry that used the methods of calculus and constructions and also there are not indicated the possible interdisciplinary relations, what cause for the students many difficulties at effective combination and using of knowledge from various areas of mathematics in solving a particular geometrical problem.
- The students prefer methods of calculus in geometry (arithmetic and algebraic approach) in comparison with methods of analytic geometry and vector calculus in solving the geometrical problems.
- Content of vector calculus represents for the students almost a mathematical tool for the analytic geometry, forgotten is possibility of its individual usage in solving some geometrical problems.
- Comprehension of algebraic and geometric parts of analytic geometry is isolated in students’ minds. Even though the students prefer analytic expression of the geometric figures in the solving of geometrical problems, connection between algebraic expression and geometrical representation of figure is not clear for them.

Concerning the analysis of historical and epistemological development of analytic geometry and vector calculus, the analysis of secondary schools’ textbooks, as well as frequent exploitation of different mathematical software and graphical calculators at secondary schools in France we formulated following additional statements, those are according to us one of possible backgrounds of students’ problems:

- Students of secondary schools are not sufficiently motivated by historical background of development of vector calculus and analytic geometry, as well as by possibility to apply obtained knowledge in other areas of mathematics, physics, informatics and in practice.
- Minimal using of information and communication technologies at the teaching of geometry decrees students’ experiences in work with geometric figures and their complete view on geometry.
5. **Proposals of possible problems’ solutions and compilation of collection of tasks that are leading to enhancement of the teaching of analytic geometry and vector calculus at Slovak secondary schools with specific recommended strategies of their solution.**

Following the analysis of actual Slovak and French secondary schools’ textbooks and also the consideration of results from the content analysis of students’ works (part 4 of this paper) we arranged set of tasks that has objective to improve the teaching of analytic geometry and vector calculus at Slovak secondary schools. This collection includes 113 tasks. With majority of tasks we present their solution or instruction for solution. In our tasks we use at the same time Slovak and French strategies as their useful combination is condition able to improve and to increase the efficiency of teaching the analytic geometry and vector calculus at secondary school.

6. **Verification of efficiency of selected tasks from our tasks’ collection by comparing the abilities and capacities of students that took part in experimental teaching (with using of our propositions and strategies) and students that participated in the teaching with using of the methods and processes, which are typical for Slovak secondary schools.**

The efficiency of tasks and strategies that we proposed in our collection of tasks, we tested by an experiment which should have verify the validity of hypothesis $H$ listed above. In this experiment we compared products of students those participated in experimental teaching with French methods and strategies (31 participants) and students that took part in teaching with typical Slovak methods and strategies (28 participants). Conclusion of application phase indicates better results in the experimental teaching with the tasks from our tasks’ collection that have the origin in French secondary schools.

At the same time, results of experiment gave us another proposals for next research. In the first place, we prefer to evaluate our experiment by the help of statistical software C.H.I.C., which permits to analyze students’ works in more detail and which underlines relations between particular variables in experiment. In the next phase of research we plan:

- to realize an experiment with more tasks from our tasks’ collection,
- to realize an experiment with more students,
- to give more time for experimental teaching,
- to repeat our experiment later in term of verification stability of obtained knowledge.
REFERENCES


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