EMBODIMENT AND A-DIDACTICAL SITUATION

IN THE TEACHING-LEARNING

OF THE PERPENDICULAR STRAIGHT LINES CONCEPT.

Autoreport by

Giannamaria Manno

ADVISOR: PROF. FILIPPO SPAGNOLO
Abstract

My research was born from the idea that difficulties and problems students of different grades have, mainly come from language and, in general, from the formal aspect of mathematical concepts.

It appeared extremely important to me to consider two sides apparently divergent

- The specific quality of mathematics and its own language
- The role of the context (space, time, people) in communicating mathematics

Do not forget that language is not only a source of trouble; it is also a necessary player in every learning process, this point is really discussed in recent works about mathematics education (Mayer, Radford, Duval).

My personal idea is that, in order to make easier the mathematical communication, it is necessary to create a proper context.

The theoretical idea I go with is the one from Guy Brousseau who in “Theory of Situation” defines the milieu: the environment where the student and his knowledge building process happen.

The choice of this theoretical framework is due to this statement:

Learning-teaching process can follow two different ways, the first one is a well known one, based on frontal lessons that are the traditional way to make people learn contents, the second one refers to a way of learning based on the emotional side (that is going to be discussed in chapter 4 about neuro-sciences) that teachers difficultly control but, on the other side, allows the building of cognitive-conflict based situation (chapter I).

We built an a-didactical situation where the relation learning-teaching into the knowledge-pupil-teacher triangle is controlled and analysed in connection with the outside environment and the emotional sphere of the student.

But I did not use only the Theory of the situation; this research aims to link two different theoretical frameworks:
The theory of the situation that structures the a-didactical situation and has a methodological control role.

The embodiment theory and neuro-science theory regarding the body experience learning, that leads to the process of creating metaphors and learning into an emotional context.

Once again I have to say that the theory of situation comes at a methodological level, the play context is built according to the theory, this play choice has been made with a look at this theory. But in this scenario another element is crucial, the play situations has also some corporeal times, so that also embodiment theory and its approach is very important as well as a theoretical reference and a way to analyse data.

It is possible to link these two theories?

We can probably say that the link between the two theoretical frameworks is that the student is the only player of his knowledge process; by the “devolution”(chapter 1) act according to the theory of the situation, by his own senses, brain and mind according to the embodiment theory (chapter 4).

What I tried to build recalls a book “cognitive space of action production and communication” by F.Arzarello\(^2\) 2004 whose basic elements are:

- The body and the brain;
- Physical world;
- Cultural environment\(^3\);

Here we can find culture, sense and motion related experiences (embodiment), languages, representations, signs and objects such as pens and computers. But Arzarello does not include in his job and approach the theory of the situation.

What does embodiment mean?

The concept of Embodiment is relatively new within the field of mathematical education, I would like to clarify, according to Lakoff and Nuñez, the use of the term in this thesis, and distinguish it from other notion concerning the role of the “physical” and “concrete” in mathematics learning. Embodiment is not simply about an individual’s conscious experience of

\(^1\) Teoria dei Costrutti Personali di G.A Kelly (1955)

\(^2\) F.Arzarello, *Mathematical, landescapes and their inhabitants: Perceptions, languages, theories*, ICME 10, 2004

\(^3\) In learning processes we find that many things interact and link to and within our body
some bodily aspects of being or acting in the world. Embodiment does not necessarily involve conscious awareness of its influence. Nor does embodiment refers to the physical manipulation of tangible objects, or to the virtual manipulation of graphical images and objects instantiated through technology. Although there is a relation between such experiences and the technical concepts of embodiment, and an embodiment perspective does not constitute a prescription for teaching in a “concrete” way. Similarly, although embodiment may provide a theoretical grounding for understanding the teaching and learning of “realistic” or “contextualized” mathematics, it is not directly concerned with “contextualization” or “situatedness” in subject matter teaching. Rather, embodiment provides a deep understanding of what human ideas are, and how they are organized in vast (most unconscious) conceptual systems grounded in physical, lived reality.

Some other important elements I consider are the conceptual metaphors. They are “mapping” that preserve the inferential structure of a source domain as it is projected onto a target domain. Thus the target domain is understood, often unconsciously, in terms of the relations that hold in the source domain. For instance, within mathematics, Boolean logic is an extension of the container scheme, realized through a conceptual metaphorical projection of the logic of containers. So a mathematical concept is build via physical experience, and later unconsciously mapped to a set of abstract mathematical concepts.

The “projections” or “mappings” are not arbitrary, and can be empirically studied and precisely stated. They are not arbitrary, because they are motivated by our everyday experience, especially bodily experience, which is biologically constrained. Unlike traditional studies of metaphors, contemporary embodied views don’t see conceptual metaphors as residing in words, but in thoughts.

When facing the problem of the comprehension of the concept of “being perpendicular”, through body experience, it is a duty to take in account the biological laws on human beings’ perception of the concepts of “being vertical” and “being perpendicular”.

To better understand this point the thesis provides an analysis of the Vestibular System (A.Berthoz) inside whom there are some receptors sensible to gravity direction.

Gravity can indeed be measured by specialized receptors: the otoliths; gravity is an external reference system, a “plumb line” the body refers to in a geocentric reference system.

The vestibular system is a very important “egocentric” reference tool that allows the perception of the plumb line model.

Mittelstaedt (1995/96) would say that we needed a new sense to add to the ones involved in gravity vertical perception, he discovered some neural receptors placed in the stomach that react
to gravity (he recently ended up saying that these structure were rather placed on kidneys or blood system).

The experimental work has been led within the S.P.O.R.A. project, the school involved were at a quite high degree of risk (based on social and economic indicators). The students involved were from 3 to 11 years old (first grade schools, scuole elementari).

The field research has required some answers on pupils’ concepts of perpendicularity:
1. Do pupils have the inner model of the plumb line?
2. Does the misunderstanding of the notion of perpendicular come from a linguistic misunderstanding?

The quantity analysis has been done with the Chic software, the quality one according to the theory of the situation.

Final considerations have been done by the process of metaphors building trough bodily activities that pupils have joined.

The aim of the thesis

The idea of a research on the development of the concept of perpendicular straight lines was suggested by the requirement of “looking at the geometry” and stimulated by the result of a text proposed in a secondary school. I have then selected as the object of this research the setting of the concept of perpendiculararity, convinced that this simple concept has a fundamental role in the comprehension but also in the build up of geometry as an operation of thought and language.

We start from some consideration to delineate the aim of the research:

- Obstacles to the correct setting of the concept are probably active in an early stage of the psycho-physical development of the subject;
- Many student understand the concept of perpendicular straight lines as verticality (language misconcept)
- Didactical interventions at various levels are generally inadequate and shallow, and they look like a lesson than built a concept.

The aim of this research is to find out different notions or ideas of the concept of perpendicularity that students have, whether if they are spontaneous or coming from teachers.

The very first question my job is based on is:
Do students have an inner model of perpendicularity between lines regarding whom the notion of perpendicularity is the same as uprightness (vertical)?

From this point of view the reference model would be the plumb line, which is linked to the concept of vertical instead of the notion of perpendicular.

It interesting to read perpendicular and vertical definitions’ given in maths dictionaries. These definitions are taken from an Italian dictionary (Rizzoli) and literally translated into English:

- **Vertical**: late Latin, *verticalis*, that goes from a top to a bottom, vertical is any line that follows the plumb line direction. Any plane that contains a vertical line is itself a vertical plane.

- **Perpendicular**: from Latin, *perpendicularum*, plumb line, it comes from hang, exactly weigh.

We can assume from the above words that perpendicularium means plumb line, but its direction does not describe the concept of perpendicular but the vertical one. In every day language indeed perpendicular is an adjective that refers to something that has the direction of the plumb line, a synonymous of vertical. The following analysis takes in great account what we have said until now, and allows to assume that one of the obstacles in learning maths is the closeness of its language and every day language such as the two words perpendicular and vertical. Often students compound different words, they take words from natural language and use them in a maths context without changing their meaning or, vice versa, they use maths words with their daily language meaning.

The aim of this research so is to investigate about the conceptions of pupils to the point of perpendicular straight lines. It’s very interesting to analyse these phenomena and search some answers to these questions:

- Have pupils the implicit model of plumb line? Are the difficulties of understanding the concept of perpendicular straight lines related to difficulties of linguistic kind?

**Theoretical framework**

The theoretical framework that I assume is the theory of situation and the point of view of the Theory of Embodiment in the particular side of building metaphors, as we can read in this schema:
In particular, I stress this concept: we built an a-didactical situation where the relation learning-teaching into the knowledge-pupil-teacher triangle is controlled and analysed in connection with the outside environment and the emotional sphere of the student.

But I did not use only the Theory of the situation; this research aims to link two different theoretical frameworks:

- The theory of the situation that structures the a-didactical situation and has a methodological control role.
- The embodiment theory and neuro-science theory regarding the body experience learning, that leads to the process of creating metaphors and learning into an emotional context.

Once again I have to say that the theory of situation comes at a methodological level, the play context is built according to the theory, this play choice has been made with a look at this theory. But in this scenario another element is crucial, the play situations has also some corporal times, so that also embodiment theory and its approach is very important as well as a theoretical reference and a way to analyse data.

**Relization of the aim**

On the first stage of the research we gave students an open test made of four questions. To be sure of the independence of their answers from teaching or research stimuli, students had been
told that their answers would have helped teachers throughout their job and would have been evaluation-free.

That has been a winning strategy because it has made students free from any emotional impact. All the students involved joined the research, a very positive result itself, considering that schools involved were all at risk (so to say, schools were dealing with a difficult social and economic context).

Further more we used cameras to shoot students over the period of research, and this has been fully accepted.

On the second stage we created an a-didactical play situation, called play-path where students had to play Tom and Jerry characters.

Teacher used the gym to build paths and ways as they were shown in the test and asked students to find out where Tom and Jerry should be in order to let Tom catch Jerry.

Students had so been able to experiment that the concept of perpendicular is not linked to the plumb line model but to the concept of minimal distance.

We followed all the stages implied in an a-didactical situation:

1. action situation
2. formulation situation
3. validation situation
4. institutionalisation situation

**Hypothesis**

The hypothesis I start from is:

**H1** Vertical and perpendicular are synonymous to students (language misconcept)

**H2** If teachers formalise the concept of perpendicular (in a maths way) students become acquainted to the concept as long as they use in a proper way the concept of the plumb line and are able to adapt it to the new situation. If students do not break this epistemological obstacle (implicit model) they will not be able to solve a given problem where the reference system is not a line crossing the centre of the earth.

**H3** To build particular “milieu” in consideration od the Theory of can be a contribute to create the didactical way more oppartune to the correct and
persistent formation of the concept. Is possible to build a context were the perpendicular straigh lines are conceptualizated as minimum distance?

We can easily think that obstacles hard to overtake are born in a very early time of psychic and physic development, and teaching action produces rather an instruction than a conceptualisation.

**The experimental work**

This experience is part of the S.P.O.R.A. project whose leading school is the D.D. Ferrara. It is worth to shortly analyse the socio-economic context before moving on. The “centro storico” district of Palermo includes old markets and places such as Ballarò, Vucciria and Capo. Inhabitants of this area often live a deep disease situation. Children’s parents often do not have a regular job, and when they eventually find one, it is a low qualification job. The analphabetic level, whether primary or secondary, is high (centres EDA to get primary and intermediate school degree have been running for years in this area).

There is also a consistent percentage of foreign people, from Africa, India and China too. Schools involved in this research project have shown throughout years the highest level of scholarship lost in Palermo neighbourhoods.

The following scheme resumes students and teachers involved in the project.

<table>
<thead>
<tr>
<th>surname</th>
<th>name</th>
<th>Teaching subject and place</th>
<th>Grade</th>
<th>Foreign students</th>
<th>origin</th>
<th>Students with special requests</th>
<th>Number of students in the class, section or schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bua</td>
<td>Sonia</td>
<td>Childhood school</td>
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<td>50%</td>
<td>Various</td>
<td>Yes</td>
<td>21</td>
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<tr>
<td>Rao</td>
<td>Margherita</td>
<td>Maths and logical</td>
<td>4° year primary</td>
<td>1</td>
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<td></td>
<td>31</td>
</tr>
<tr>
<td>Ruvolo</td>
<td>Francesca</td>
<td>Maths and logical</td>
<td>2° year primary</td>
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<td>Bangladesh</td>
<td>Many</td>
<td>24 23</td>
</tr>
<tr>
<td>Troia</td>
<td>Maria</td>
<td>Linguistico/espressivo</td>
<td>1° year primary</td>
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<td>Gallina</td>
<td>Rosetta</td>
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<td>Tabbi</td>
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<td>Maths and</td>
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<td>10</td>
<td>35(scheduled teaching )</td>
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In the first phases of the experience the teachers and I proposed a text, where two important animals are the protagonist: Tom and Jerry. They acts in an important cartoon, that is very famous for all Italian pupils.

The test consists of four questions that require drawing a line that joins Tom and Jerry position when Tom is sure to catch Jerry.

That line is the one that crosses the point where Tom is and is perpendicular to the line where Jerry walks in.

The first question of the four is quite easy because in this case vertical and perpendicular are the same.

Second question requires finding out the perpendicular line when Jerry walks on a slanting line (changing the reference system from the first question).

Third question is similar to the second one but it is open to two different solutions; and this is quite interesting because children think (didactical contract) that teachers cannot give twice the same problem and tend to answer in different ways.

The last question is an open problem because it is about finding the perpendicular to a curve crossing it in a point.

To analyse the text I realize an “analysis a priori”, where I consider all the answers that the student can be do.

The second phases of the research is the moment of the play-way.

In the gym there are paths similar to the ones in the given exercises.

- matter used: sellotape, paper sheets with the path to go through, rope and string
- teams: every team has five members, three of them have been given the role of tom and jerry and a time keeper, the two left have an observing role
- time-in of the play: one of the children keeps the time clapping his hands. The pink child goes a step forward every time he hears a clap. The yellow child waits keeping his position as in fig. (make children repeat the path several times and
give them a string telling how to use it, or say that it can be useful to solve the problem)

The pink point shows the beginning position of the child playing Jerry, A, B, C indicates different beginning position of Tom-child, as they change every time the play begins.

- stage 1 explanation and demands: teachers explain the game, simulating every step and the rules
- demand: two of you play Tom and Jerry, Tom sleeps waiting for Jerry to arrive, Jerry walks. In what position are Tom and Jerry the closest? Left members of the team have to draw the game in order to understand what strategy suits the best. Who finds the winning strategy wins the competition. How can Tom realize the best moment to jump and catch Jerry? Find out a method that allows Tom to catch Jerry wherever the mouse is.
- Stage 2 (action situation): different teams draw on a sheet the play situation finding the right strategy
- Stage 3 (formulation) different teams socialize sharing their strategies
- Stage 4 (validation) teachers ask to express the found strategies, some shared conjectures are required in order to become theorems, in this stage it could be useful to analyse the use of the string.
- Stage 5 (institutionalisation) a new theorem is born, as a solution or winning strategy, shared by everybody.

Resume of the phases of the research:
- To suggest an open text;
- To realize an “a-didactical situation”
- To analize the text;
To realize the “Analisi a priori”;
- Quantitative (chic software) and qualitative analysis of the data

**Evaluation of the experiment**

The experimental data of the research are interpreted by qualitative and quantitative analysis. In particular, the results of the text are interpreted with the statistical software chic and with qualitative analysis. For the nature of the play-way, it is possible only a qualitative analysis. I report here only a resume of the principal results.

**Quantitative analysis of the text:**

Demand: children generally have been able to answer and give explanations, the analysis of the entire body of tests, so to say their percentage, would let us think that finding the minimal distance between two points is the same that finding a vertical line crossing one of the two given points.

After a first glance on children explanations, we can assume that the concept of vertical is seen through the lens of daily experience of gravitation.

Children know that if they let an object falling, it goes towards the floor, thus if Tom jumps when Jerry is under him, he will catch him. Students consider the vertical position between Tom and Jerry a winning strategy. To enhance our opinion, we can look at some children explanations to what they have done:

“Because the mouse walks and at a point all of a sudden the cat jumps on him”

“Because it is easier”

“Because Tom was ready and jumped at the right time”

“Because Tom jumps and catches him”

“Because the cat is in line with the mouse and it is easy to catch him”;

There have been children that understood that uprightness (vertical) was not the right answers for all the questions, and that was better to make Tom jump when Jerry was the closest “the cat waits for the mouse to be closer”. Also very interesting are children explanations about the position choice of Tom and Jerry, they have not been deceived by the idea that if Tom jumps following the vertical line he will go faster and all of a sudden he will catch Jerry, children rather have an idea (they are not yet presented the concept of perpendicular) of the concept of minimal distance. Here we have some of the children explanations:

“because he is closer”

“because the cat waits for the best time”
“because Tom has been waiting for Jerry to come closer and he caught him”
“because Tom wants to catch Jerry and when he is closer he catches him”

Other children did not follow the two ways described above, but thought that, being Jerry on the top of the roof, he was closer to Tom, or they found a hiding place for Jerry (answers where they connects the blue point to the horizontal segment C6). Many children showed an emotional commitment during the test, thing that was not foreseen, being close to the Jerry point of view or to Tom “because he was hungry”. Other students misunderstood the demand, explaining that the essential condition to get Jerry is to be well hidden, to succeed in catching the mouse Jerry does not have to be conscious of Tom being there. Other children answered that Tom would have caught Jerry only if “the mouse had been slow on his run”.

Quantity analysis of 1-2 grade primary school (age 5 to 7) tests

The quantity analysis has been done with Chic software on a sample of 33 students, not statistically meaningful, but useful in order to suggest future research hypothesis. After an accurate a-priori analysis we chose to put together right and wrong ideas. The analysis of the similarity reveals that children ideas are split into three directions: Strong similarities are:

- Able to find out the vertical line in exercise 1 and to guess vertical in exercise 3
- Reveals that the correct strategies uses in the 3-4 exercise is not a general competence infact the student uses a wrong strategies in 1-2 exercise.
- The last similarity group shows that when children have fragile correct ideas they are not able to make them general and to use them in different contexts.

The analysis of the implicative graph reveals that children ideas are split into three directions

- If a child finds out the perpendicular line in exercise 1 he will have some correct ideas in left exercises but not always he will follow the correct strategy;
- If a child follows once a wrong strategy he will follow it also in a different context
If a child finds out the perpendicular line in exercise 3 he can also understand the minimal distance between Tom and Jerry in exercise 4. If we consider answers children gave in exercises 3 and 2 we can assume that even if they are in front of similar questions they give different answers. To explain this we could say that the problem is in the number of solutions, only one possible in exercise 2 and two possibilities in exercise 3. Children follow the didactical contract and think that it is not possible that two exercises have the same solution, especially if they do not understand that there are two possible choices to solve exercise 3. Why should teachers give two exercises that are almost the same?

Quantity analysis of 3-4 grade primary school (age range: 8-9 years) and 1 intermediate (10-11) tests

The data analysis has been done on a 73 students sample using Chic software. The different social-cultural context has made possible to cling together data from primary grades and first intermediate.

The similarity analysis reveals that children ideas can be shared in three groups:

- Many student consider perpendicular straight lines and verticality as the same concept, they stress the verticality for indicated the perpendicularity.
- Some students have the correct idea of the perpendicular line, but it is only an idea no a mathematical concept;
- Some answers given by the students reveals that children draw a line that connects the cat to the highest, and closest to them, mouse point.

From the graph we can imply that:

- children have the same model of vertical and use it in different exercises. (The model of plumb line is very strong and childrens uses it in different context);
children that guess perpendicularity in exercise 4 do not get an implicit and inner model of vertical.

To muse is possible to think that the implicit model of verticality is very strong in the student, and those student that haven’t this inner model are not able to identify the mathematical concept of perpendicularity. They possibly knows that perpendicular straight lines are nor verticality lines but they haven’t the formalization of the concepts.

Play-path analysis

During the first path teams have found the “right” point thanks to the concept of vertical. They in fact used the following expressions:

“is under Tom”

“it is straight”

This can make us think that they still do not have a dominant concept of closeness between their point and Tom, but they rather have a strong idea of vertical; although a student tried to talk about it, the rest of the team-mates did not pay very much attention to it. It was still a rough guess.

While doing the second path one team kept following the concept of vertical repeating the word under. Other teams, also not forgetting the vertical concept, have found as more relevant the “in front” position of cat and mouse, thing that makes easier to see each other (here more easily seen than in a written test): to prove that, they often use the word “he sees better” and criticise the curve on other teams paths.

The idea of closeness, even if in a certain way guessed, was still not fully expressed and understood in its importance.

During the third path, the idea of closeness as the most important criteria to solve the problem starts to reveal its importance, also thanks to some stimulus-questions.
In order to help students to find the winning strategy they had been given a string without being
told of its usefulness. Students repeated their ways using this string. One edge of the string had
been given to “Tom” the other one to “Jerry”, they had also been told to keep this string always
straight, thus “Jerry” had to wind up and down the string to keep it always straight. This has been
essential to make them realize that the string was shorter in the closest points and longer in
points very far from each other.

Students have had group discussions about winning strategies (validation) and ended up on an
agreement: the “right” point to catch Jerry was the closest and this point was the perpendicular
concept itself.

The most successful theorem was: “the perfect time for Tom to catch Jerry is when the mouse is
the closest to the cat not when he is under the cat”

Finally the team that followed the correct strategy in both the form and the game paths won the
“competition”.

**Conclusion**

Third chapter tells us how the experiment bears pupils to succeed in finding the right theorem;
when pupils understand that the concept of “being perpendicular” has to be related to the
distance one, they find the way to solve the problem, overcoming the obstacle of the similar
meaning of the words “perpendicolarità” and “verticalità”.

The hypothesis we start from is the one about the uniqueness of mathematics and its language;
otherwise fourth chapter describes us the subjective vertical and suggests that difficulties we find
in defining the concept of perpendicularity are strictly linked to our body and brain (we do have
gravity receptors in our abdomen). We can easily assume that pupils’ problems can have
different reasons.

It has been really interesting evaluating the results coming from the embodiment experience; this
experience has made clear that there are different set of metaphors, some of them are related to
the perpendicular concept and some others are related to the vertical concept. The play-path has
helped a lot in this achievement.
When using the embodiment theory we also receive a positive feedback on the effectiveness of the theory of the situation, but these two theories are fully valid alone, they do not need each other positive feedback to show that they are both true.

Pupils have built a mental image that links the concept of perpendicularity to the plumb-line model, there are several reasons for this: the similar meaning of the two words, as we said above, the inner knowledge of pupils, a personal experience of the gravity concept, the way our body works; but at the end this strategy is successful in some particular cases.

The play-path undermines this mental image and requires a new one that is finally successful in every case, not only in some specific cases.

We have seen that after this experience pupils link the concept of perpendicular to the one of minimal distance.

The embodiment theory proves this result and explains it (new metaphors are born), and it is the experience itself that let pupils link the right metaphor to the right mathematical concept.

In my opinion it has been determining the environment we provided in order to experiment these mathematical concepts, it would be really interesting to repeat a similar experience with other mathematical concepts.
My publications

http://dipmat.math.unipa.it/~grim/quadernosuppl10.htm

http://dipmat.math.unipa.it/~grim/21_project/21_brno_03.htm
References

- Bart Kosko, 2000, “Pensiero Teoria e Applicazioni della logica fuzzy”, c.ed, Baldini e Castoldi
- Boero P., DeLuca C., 1996, La letteratura per l’infanzia, Bari, Laterzia
- Boero, Garuti, Le disequazioni come ambiente di sviluppo dei concetti di variabile e funzione, in vista dell’analisi matematica, Sviluppo del testo di sfida-16.23/V/01
- Carl B. Boyer, Storia della matematica, Mondadori, Milano 1980
- Cutrera, Mustacci, 1987, La formazione del concetto di perpendicolarità, Grim
- Cutugno P., Le concezioni degli allievi di scuola elementare sul triangolo, Tesi di Laurea consultabile all’indirizzo http://math.unipa.it/~grim
- Changeux Jeanne Pierre, L’uomo di verità, trad. Alessandro Serra, Feltrinelli, Milano
- D’Amore Bruno, Elementi di Didattica della Matematica, Pitagora Editrice Bologna, 1999
- De Mauro T, Minisemantica, Editori LaTerza, Bari, 2000
- Eco Umberto, 1975, Trattato di semiotica generale, Bompiani Editore, Milano
- G.R.I.M., Quaderni di ricerca in didattica N.3, Palermo 1982
- G.R.I.M., Quaderni di ricerca in didattica N.8, Palermo 1999
- G.R.I.M., Quaderni di ricerca in didattica N.9, Palermo 2000
- G.R.I.M., Quaderni di ricerca in didattica N.8, Palermo 2003
- Lakoff G., Núñez E.R., 2000, Where Mathematics comes from
- Maier h (1993), Problemi di lingua e comunicazione durante le lezioni di matematica, La matematica e la sua didattica
- Maier h (1993), *Il conflitto tra lingua matematica e lingua quotidiana per gli allievi*, La matematica e la sua didattica
- Maraschini, Palma, *Format*, Paravia
- Pesci A., Tesi Master, cap I-II, vedi sitografia
- Radford, 2002, “*The seen, the spoken and the written. A semiotic approach to the problem of objectification of mathematical Knowledge*”. For the learning mathematics, 22(2)14-23)
- Spagnolo Filippo, (1984), *Errori matematici un occasione didattica. L’insegnamento della matematica e delle scienze integrate, I*
- Spagnolo Filippo e Ferreri Mario, *l’apprendimento tra emozione ed ostacolo*, lavoro eseguito nell’ambito del contratto C.N.R n. 9001293CT01

Sitetografia:
- [www.dipmath.math.unipa.it/~grim](http://www.dipmath.math.unipa.it/~grim)
- [www.dipmath.math.unipa.it/~grim/tesiM-pesci04](http://www.dipmath.math.unipa.it/~grim/tesiM-pesci04)
- [www.ipnosicostruttivistavista.it](http://www.ipnosicostruttivistavista.it)
• www.uniurb.it
• www.mfnunipmn.it